Boundary Layer Transition: Freestream Turbulence and Pressure Gradient Effects

E. R. van Driest* and C. B. Blumer†
North American Aviation Inc., Downey, Calif.

A theory of boundary layer transition is presented which includes the effects of freestream turbulence and pressure gradient. The basic notion underlying the development concerns the remoteness of the boundary layer vorticity from the surface. The theory agrees well with experimental data. The laminar boundary layer solutions of Pohlhausen and Falkner-Skan are used to calculate the effect of pressure gradient on transition. Engineering formulas are presented, and one example is calculated showing the application of the theory to a body of revolution. The theory also can indicate the effects of Mach number and heat transfer.

Nomenclature

C_p = pressure coefficient, $(p_x - p_\infty)/\frac{1}{2}\rho_\infty u_\infty^2$
l = length in boundary layer
m = Falkner-Skan exponent
p = pressure
Re_δ = Reynolds number based on boundary layer thickness, $u_\delta\delta/\nu$
Re_x = Reynolds number based on boundary layer length, $u_\delta x/\nu$
Tr = vorticity Reynolds number value at transition $[y^2(du/dy)/\nu]_{\text{trans}}$
u = velocity in direction of flow
u' = root-mean-square velocity fluctuation
u_δ = freestream velocity at edge of boundary layer
x = distance along surface
y = distance normal to surface
δ = thickness of boundary layer
η = $(y/x)(u_\delta x/\nu)^{1/2}$
λ = eddy size in freestream
Λ = Pohlhausen parameter $(-\delta^2/\mu u_\delta)dp/dx$
μ = fluid viscosity
ν = kinematic viscosity
ρ = fluid density
τ = fluid shear stress

Introduction

Although the process of transition from laminar to turbulent flow has been the subject of intense research for many years, its mechanism as yet is not understood fully. In the present paper an attempt is made to contribute some further ideas on the mechanism of laminar flow breakdown.

The approach followed here is to assume that breakdown of the laminar flow occurs whenever the local vorticity (shear) within the boundary layer is sufficiently remote from the surface. Using this criterion and following the method of Taylor for introducing disturbances into the flow, a boundary layer transition theory is developed which takes into account the effects of freestream turbulence and pressure gradient and can be applied to practical problems of flow about bodies.

Formulation of the Transition Criterion

The forementioned criterion for transition may be formulated by considering that the ratio of the local inertial stress, $\rho l^2(du/dy)^2$, to the local viscous stress, $\mu du/dy$ [viz., $\rho l^2(du/dy)/\mu$], must reach a limiting value somewhere in the flow in order for transition to take place. In these expressions ρ is fluid density, μ is fluid viscosity, l is a length, and du/dy is the velocity gradient at distance y from the surface. When it is assumed that length l is proportional to y, the criterion becomes

$$[(y^2/\nu)(du/dy)]_{\text{trans}} = \text{const} = Tr \quad (1)$$

which is called the vorticity Reynolds number at transition. In terms of the local shear stress τ, Eq. (1) may be written as

$$(y/\nu)(\tau/\rho)^{1/2} = Tr^{1/2} \quad (2)$$

which is the form previously applied to the "transition" from the viscous sublayer to the turbulent flow in the solution of the turbulent boundary layer with mass transfer.[1]

The vorticity Reynolds number profile for a Blasius boundary layer is shown in Fig. 1.‡ It is noted that the limiting value Tr of the vorticity Reynolds number function $y^2(du/dy)/\nu$ will occur in an undisturbed laminar boundary layer at an appropriate length Reynolds number, say Re_x, which must be determined from experiment. Hence, any modification of the laminar Blasius profile, as with pressure gradient, compressibility, heat transfer, or mass transfer, will alter the vorticity Reynolds number profile, which correspondingly will alter the length transition Reynolds number. Also, superimposed disturbances such as freestream turbulence, surface roughness, surface waviness, or Tollmien-Schlichting

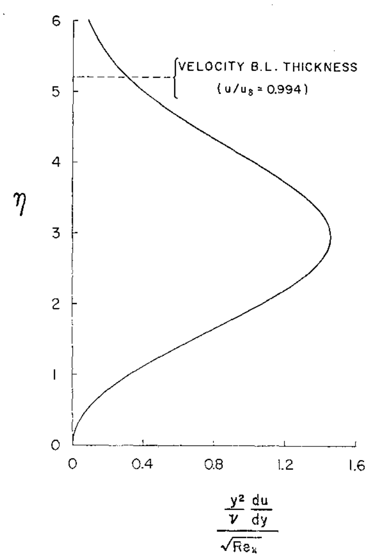

Fig. 1 Vorticity Reynolds number profile

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* Director, Space Sciences Laboratory, Space and Information Systems Division. Member AIAA.

† Research Scientist, Space Sciences Laboratory, Space and Information Systems Division. Member AIAA.

‡ It is noted that this function was considered by Rouse as a stability parameter in Ref. 14.

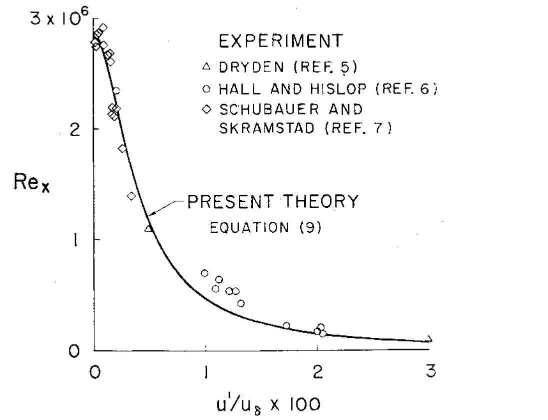

Fig. 2 Effect of freestream turbulence on transition

waves will increase the value of the vorticity Reynolds number function over that calculated for an undisturbed profile with a resulting reduction in transition Reynolds number.

It is important to observe that in this theory momentary separation of the flow is not required in order for transition to occur; in fact, separation is important only to the extent that it increases the vorticity Reynolds number.

According to Fig. 1, initial laminar breakdown should occur at a location within the boundary layer which coincides with the maximum of the vorticity Reynolds number function. For the Blasius profile, this maximum occurs at $y/\delta = 0.57$, where δ is the velocity boundary layer thickness. Therefore, a first check on the validity of the transition criterion assumed in the foregoing is the observation by Klebanoff and Tidstrom[2] and later by Kovasznay, Komoda, and Vasudeva[3] that high-frequency bursts representing local flow breakdown originate at $y/\delta \approx 0.6$ for an incompressible flat plate boundary layer, which coincides strikingly well with the maximum position of the vorticity function.

Determination of Effects of Freestream Turbulence and Pressure Gradient

The effects of freestream turbulence and pressure gradient on transition now may be determined as follows. It first is assumed following Taylor[4] that freestream pressure disturbances distort the boundary layer in accordance with the Pohlhausen theory. Thus, upon calculation of velocity profiles in terms of the Pohlhausen parameter Λ, one readily can demonstrate that, approximately,

$$\frac{[y^2(du/dy)/\nu]_{\max}}{Re_\delta} = \frac{Tr}{Re_\delta} = A + B\Lambda \qquad (3)$$

where Re_δ is the Reynolds number based on freestream conditions and thickness δ of the boundary layer. Writing Λ as the sum of its mean value $\bar{\Lambda}$ and fluctuation Λ' from the mean, Eq. (3) becomes

$$Tr/Re_\delta = A + B(\bar{\Lambda} + \Lambda') \qquad (4)$$

or, since $\Lambda = -(\delta^2/\mu u_\delta)(dp/dx)$,

$$\frac{Tr}{Re_\delta} = A + B\frac{\delta^2}{\mu u_\delta}\overline{\left(\frac{dp}{dx}\right)} + B\frac{\delta^2}{\mu u_\delta}\left(\frac{dp}{dx}\right)' \qquad (5)$$

where p is static pressure, u_δ freestream velocity, and x distance along the flow. Now, since fluctuating quantities are being dealt with, take the root-mean-square value and let, according to Taylor, $(dp/dx)' \sim \rho u'^2/\lambda$, in which u' is the root-mean-square velocity fluctuation, and λ is a measure of the scale of the eddies in the freestream. It next may be reasoned that the eddy size must be of the same order as the boundary layer thickness δ in order to effect a significant change in the vorticity Reynolds number. Hence, substituting Taylor's relation into Eq. (5) and assuming that $\lambda \sim \delta$, there results

$$Tr/Re_\delta = A + B\bar{\Lambda} + C\,Re_\delta(u'/u_\delta)^2 \qquad (6)$$

For similar boundary layer solutions, Eq. (6) can be written in terms of Re_x; thus

$$Tr/Re_x^{1/2} = (A + B\bar{\Lambda})\eta_\delta + C\eta_\delta^2\,Re_x^{1/2}(u'/u_\delta)^2 \qquad (7)$$

where $\eta_\delta = (\delta/x)(u_\delta x/\nu)^{1/2}$. When the pressure gradient is zero, $\bar{\Lambda} = 0$, and Eq. (7) reduces to

$$Tr/Re_x^{1/2} = A\eta_\delta + C\eta_\delta^2\,Re_x^{1/2}(u'/u_\delta)^2 \qquad (8)$$

Equation (8) may be tested upon comparison with available data from Dryden,[5] Hall and Hislop,[6] and Schubauer and Skramstad.[7] Figure 2 shows such a comparison, and the theory appears to be valid. The equation with numerical constants adjusted to fit the data is

$$1690/Re_x^{1/2} = 1 + 19.6\,Re_x^{1/2}(u'/u_\delta)^2 \qquad (9)$$

When the coefficient of $\bar{\Lambda}$ is computed using the Pohlhausen fourth-degree velocity profile, Eq. (6) becomes

$$9860/Re_\delta = 1 - 0.0485\bar{\Lambda} + 3.36\,Re_\delta(u'/u_\delta)^2 \qquad (10)$$

which is plotted in Fig. 3 for various values of $\bar{\Lambda}$.

Calculations also have been carried out based on the wedge-flow solution of Falkner and Skan.[8, 9] In this solution, the potential flow velocity at the outer edge of the boundary layer is proportional to a power of the distance from the stagnation point, i.e., $u_\delta \sim x^m$. The transition equation that results from these calculations is

$$1690/Re_x^{1/2} = 0.312(m + 0.11)^{-0.528} + 0.73\eta_\delta^2\,Re_x^{1/2}(u'/u_\delta)^2 \qquad (11)$$

The first term on the right-hand side of this equation is an empirical pressure gradient term that yields better accuracy than the corresponding linear term of Eq. (7). The boundary layer thickness parameter η_δ is tabulated as a function of m in Table 1, and Eq. (11) is presented graphically in Fig. 4 for a range of m from $m = -0.0904$, the separation profile, to $m = 4.0$, an extreme favorable pressure gradient.

For the two-dimensional stagnation point, a direct comparison may be made between the Falkner-Skan ($m = 1$) and Pohlhausen ($\Lambda = 7.052$) solutions. The results of this calculation are shown in Fig. 5. It should be remembered that either solution will revert to Eq. (9) for the zero pressure gradient case. Hence, one may conclude that, for favorable pressure gradients up to and including the stagnation point value, the agreement between Falkner-Skan

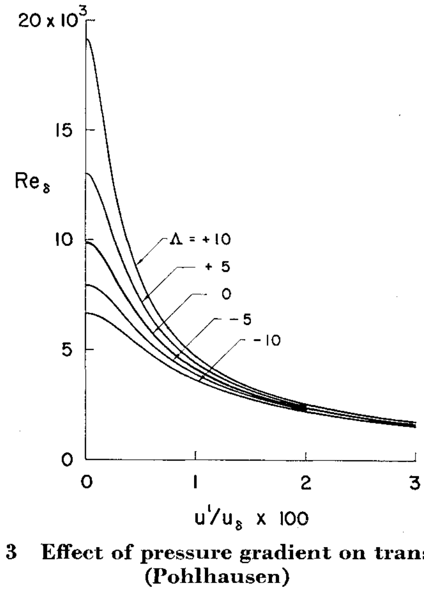

Fig. 3 Effect of pressure gradient on transition (Pohlhausen)

Table 1 Boundary layer thickness parameter as a function of Falkner-Skan exponent

m	η_δ
−0.0904	7.36
−0.08	6.48
−0.04	5.66
0.0	5.18
0.1	4.46
0.5	3.22
1.0	2.56
2.0	1.95
3.0	1.63
4.0	1.43

and Pohlhausen will be as good or better than shown in Fig. 5.

The axially symmetric stagnation point is also a case of special interest. An exact laminar boundary layer solution for this case has been worked out by Frössling[10] and was used to compute the vorticity Reynolds number profile. The resulting transition equation for the axially symmetric stagnation point is

$$6160/Re_x^{1/2} = 1 + 11.7\,Re_x^{1/2}(u'/u_\delta)^2 \tag{12}$$

which also is shown in Fig. 5. It is interesting to observe that the difference in pressure gradient is compensated by the difference in boundary layer thickness between the plane and axially symmetric cases, resulting in a transition Reynolds number for the axially symmetric flow which is only slightly greater than for the two-dimensional flow.

Application to Flow about Bodies

As an example of the application of the foregoing transition theory to flow around bodies of revolution, the transition Reynolds number Re_x was calculated on an ellipsoid of revolution of slenderness ratio 9 as a function of transition position on the body. This body has been tested extensively in flight and in two low-turbulence wind tunnels.[11] The boundary layer properties were determined by an approximate procedure for bodies of revolution as outlined in Ref. 12, which parallels the two-dimensional Pohlhausen method. The calculation yielded the pressure gradient parameter Λ and the dimensionless boundary layer thickness η_δ, each as a function of body station. Once these quantities were known, it was a simple matter to apply Eq. (10) to compute the transition Reynolds number Re_δ based on boundary layer thickness for a given stream turbulence. From Re_δ and η_δ, follow Re_x, which is plotted in Fig. 6 as a function of body station for several turbulence levels. The ellipsoid shape and pressure distribution are shown in Fig. 6 as supplementary information. The experimental transition data of Ref. 11 also are plotted in Fig. 6. The differences between theory and experiment are not completely explainable at this writing; however, several general remarks are appropriate. First, the flight test data and the theory follow the same trend, that is, transition Reynolds number increases with increasingly favorable pressure gradient. In fact, it appears that the $u'/u_\delta = 0.2\%$ turbulence curve could be used to extrapolate the flight data with fairly good accuracy. Second, the wind tunnel data may suffer from a variation in freestream turbulence level over the velocity range of the tests. A subsequent investigation[13] in the NASA Ames 12-ft pressure wind tunnel has shown that the level of velocity fluctuation, due to sound intensity in the wind tunnel, increased by a factor of 5 between Mach 0.1 and 0.3, which coincides with the range of the ellipsoid tests. Hence, it is possible that a variation in effective turbulence in these experiments masked the effect of pressure gradient on transition Reynolds number in these experiments.

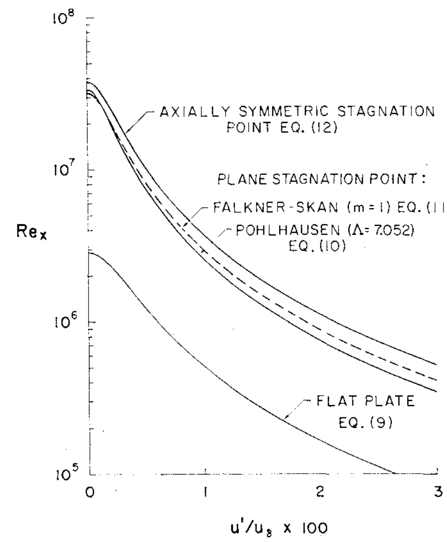

Fig. 5 Stagnation point solutions

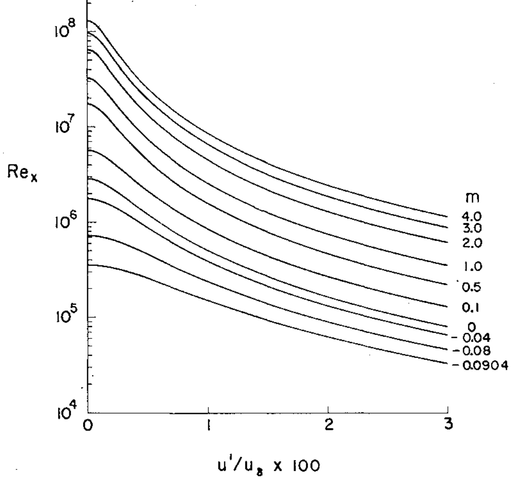

Fig. 4 Effect of pressure gradient on transition (Falkner-Skan)

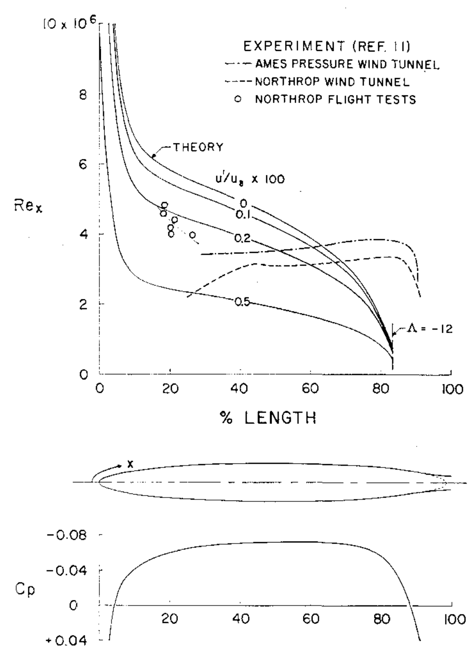

Fig. 6 Transition on an ellipsoid

Compressibility Effects

Preliminary calculations of the vorticity Reynolds number profile for a compressible laminar boundary layer shows that the maximum value of this function moves toward the edge of the boundary layer with increasing Mach number and also indicates a decrease in transition Reynolds number with increasing Mach number. Further calculations including heat transfer indicate that at constant Mach number the transition Reynolds number should increase with decreasing wall temperature. These trends are consistent with experiment.

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Some Characteristics of the Turbulent Boundary Layer with Air Injection

O. E. Tewfik*
University of Minnesota, Minneapolis, Minn.

Measurements of turbulent velocity profiles on a 2-in.-o.d. circular cylinder, aligned with its axis parallel to an approximately 110-fps air stream and with air injection into the boundary layer, are described. The injection rate per unit area of cylinder surface was uniform and equal to 0.00107, 0.00202, 0.00312 of the freestream mass velocity. By means of appropriate mass and momentum balances, the local and average skin friction and the distributions of the radial velocity component and shear through the boundary layer were determined. Study of the results yielded the following. 1) Contrary to the assumption in some theoretical analyses, the radial velocity component was not constant through the boundary layer but increased from its wall value to up to three times as much at the edge of the boundary layer. 2) The assumption that the excess of shear stress in the boundary layer over the wall-shear stress is equal to the product of the mass injection rate per unit area and the local axial velocity component agreed well with the results of measurements in the inner tenth of the boundary layer but became progressively poorer towards the edge of the boundary layer. 3) The local skin friction was found to agree with the measurements of Tendeland and Okuno but to be substantially higher than those of Mickley and Davis and of Pappas and Okuno. 4) The measured ratio of skin friction to its value with no injection agreed well with Rubesin's and Van Driest's analyses for a smooth flat plate but disagreed markedly with Turcotte's analysis. 5) Injection increased all boundary layer thicknesses and distorted the velocity profile from its typically turbulent shape for no injection, the extent of the distortion depending upon the injection rate.

Nomenclature

C_f = dimensionless shear stress = $2\tau/\rho u_\infty^2$
C_{fw} = local skin friction = $2\tau_w/\rho u_\infty^2$
C_{fw0} = local skin friction without injection
$\bar{C}_{fw}$ = average skin friction, Eq. (5)
$\bar{C}_{fw0}$ = average skin friction without injection
m = index in power law for boundary layer growth, Eq. (9)
$\dot{m}$ = air mass injection rate per unit area of outer cylinder surface
n = index in power law for velocity u in terms of y, Eq. (9)
p = pressure
q = index in power law for average skin friction in terms of Reynolds number

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* Currently Project Engineer, Corning Glass Works, Corning, N. Y. Member AIAA.